MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2017/2018**

## ETM 7156 – MOBILE WIRELESS COMMUNICATIONS

6 MARCH 2018
2:00 PM – 5:00 PM
(3 Hours)

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**INSTRUCTIONS TO STUDENT**

1. This Question paper consists of 10 pages (including this cover page) with 5 Questions and 2 Appendices only.

2. Attempt **ALL FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Please number your answers clearly.

**Question 1**

(a) With the aid of a diagram, explain the motivation and working principle behind the cellular approach adopted in modern mobile communications. [7 marks]

(b) A *blocked-calls-cleared* based cellular system has a total bandwidth of 98 MHz and each duplex channel requires 200 kHz. Assume that each user produces 0.1 Erlangs of traffic and a seven-cell reuse pattern is used. An Erlang B table is provided in Appendix 1.

(i) Determine the total number of duplex channels per cell. [2 marks]

(ii) Assume that each cell can support an amount of traffic intensity (in Erlangs) that is equivalent to 95% of the total number of duplex channels per cell. Find the number of users that can be accommodated in a cell, assuming that omnidirectional antennas are used at each base station. [2 marks]

(iii) Determine the blocking probability of the system in (ii). [2 marks]

(c) A cellular system experiences path loss exponent $n$ of 3.5, where the user equipment requires a minimum signal-to-interference ratio (SIR) of 18 dB. The cellular system is preliminarily designed to use a cluster size $N$ of 7 and omnidirectional antennas at the base stations. The SIR on the forward link can be computed using the expression

$$\text{SIR} = 10\log_{10}\left[\frac{\left(\sqrt{3N}\right)^{n}}{i_0}\right] \quad \text{(in dB)},$$

where $i_0$ is the number of interfering base stations in the first tier.

(i) Assume six cells are close enough to create significant interference (i.e., $i_0$ = 6), determine the SIR of the above system. Can the above design meet the requirement? [2 marks]

(ii) If the system designed above does not meet the SIR requirement, illustrate with a figure and through calculation, how the SIR requirement can be met by deploying sectored antennas of 120° beamwidth for 3 sectors per cell. [5 marks]

**Continued .....**

## Question 2

(a) (i) Define the three basic propagation mechanisms in mobile radio channels. [3 marks]

(ii) Discuss briefly the two major categories of factors influencing small-scale fading in both the time and frequency domains. [5 marks]

(b) The transmission tower of a base station, $T$, serves a housing area by providing digital transmission to receiver units, $R$, installed in houses, where the average receiver antenna height is 10 m. Standing in between the transmission tower and the receivers is a small hill which peaks at 30 m. The hill can be modeled as a knife-edge obstruction as shown in Fig. Q2.1. The carrier frequency is 2 GHz. Determine the knife-edge diffraction loss. The speed of light can be approximated as $3\times10^8$ m/s.

[*Hint*: The Fresnel-Kirchoff diffraction parameter is given as $v=\alpha\sqrt{\frac{2d_1d_2}{\lambda(d_1+d_2)}}$, where $\lambda$ is the wavelength and the interpretations of other symbols are shown in Fig. Q2.1. The diffraction loss can be calculated using the following equation.]

[7 marks]

$$G_d(\text{dB})=\begin{cases}0, & v\le -1\\ 20\log_{10}(0.5-0.62v), & -1\le v\le 0\\ 20\log_{10}(0.5\exp(-0.95v)), & 0\le v\le 1\\ 20\log_{10}\left(0.4-\sqrt{0.1184-(0.38-0.1v)^2}\right) & 1\le v\le 2.4\\ 20\log_{10}\left(\frac{0.225}{v}\right), & v>2.4\end{cases}$$

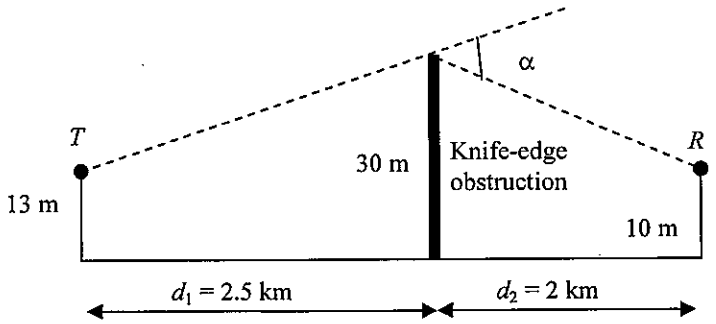


Fig. Q2.1

**Continued .....**

(c) Fig. Q2.2 shows the measured channel impulse response obtained from a channel sounding experiment on a wireless channel.

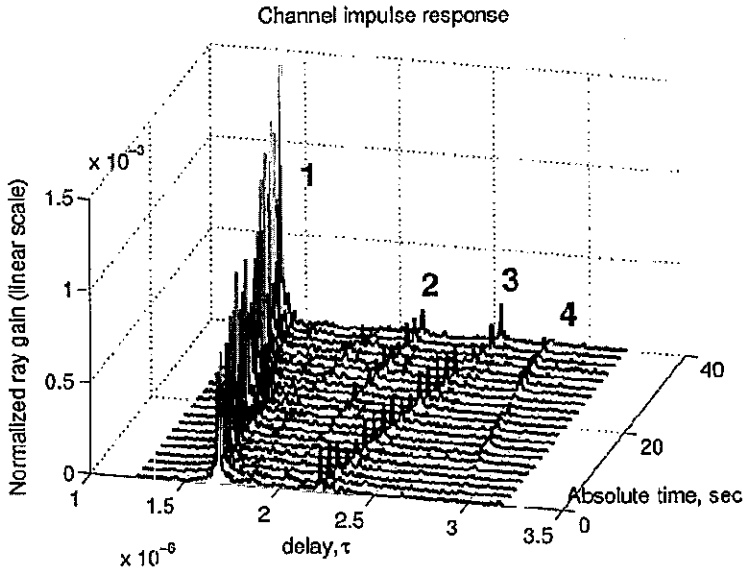


Fig. Q2.2

(i) Briefly discuss the nature of this wireless channel. [2 marks]

(ii) Can reliable signal transmission be achieved over this channel if the signal bandwidth is much larger than the channel's coherence bandwidth? If the answer is no, suggest a suitable technique to be used at the *receiver*. [3 marks]

**Continued .....**

**Question 3**

(a) Define the terms *power efficiency* and *bandwidth efficiency* of a modulation scheme. Discuss the tradeoff between them using $M$-ary phase shift-keying ($M$-PSK) modulation with $M = 2$ and $M = 8$ as an example. [5 marks]

(b) The bit error rate (BER) of a Direct Sequence Spread Spectrum (DS-SS) cellular system with power control where the received signal power of all users at the base station is the same is given by

$$P_e = Q\left[1/\sqrt{\frac{K-1}{3N} + \frac{N_0}{2E_b}}\right]$$

where $Q$ is the $Q$-function (table is provided in Appendix 2) and $K$ is the number of users, $N$ is the length of spreading code, $N_0$ is the noise power density, and $E_b$ is the average energy per bit. How many simultaneous users may be supported such that an average BER of less than $10^{-3}$ is maintained for each user? Each user is maintained at an average $E_b/N_0 = 10$ dB, and assume each user has a pseudo-random number (PN) code that is produced from an 11-bit shift register. [5 marks]

(c) (i) Briefly describe the *purpose* of estimating the impulse response of a wireless channel in practice. [2 marks]

(ii) Describe with the aid of a diagram and mathematical equations, how the *impulse response* of a time dispersive channel can be estimated using a probing signal with an impulse-like autocorrelation function. [8 marks]

**Continued .....**

**Question 4**

(a) (i) Define the two major categories of channel equalizers used in digital communication systems. [2 marks]

(ii) Sketch the structure of a linear transversal equalizer. [2 marks]

(iii) With the aid of a diagram, briefly describe the training mode and decision-directed mode used in a linear adaptive channel equalizer. [6 marks]

(b) A wireless communication channel experiences $M$ independent Rayleigh fading channels. The receiver uses *selection combining* technique. Assume that all branches have identical average signal-to-noise ratio (*SNR*), $\Gamma$. In addition, the instantaneous *SNR* of each branch follows a Rayleigh distribution. If $\Gamma = 25$ dB and a specific *SNR* threshold, $\gamma$, is defined as 20 dB, answer the following questions.

[*Hint*: The probability that $M$ diversity branches *simultaneously* have instantaneous SNR less than a threshold $\gamma$ is given by $P_M(\gamma) = \left(1 - e^{\frac{-\gamma}{\Gamma}}\right)^M$ ].

(i) Briefly describe the principle of selection combining technique. [3 marks]

(ii) The probability that a single branch has an instantaneous *SNR* value smaller than the threshold. [2 marks]

(iii) If $M = 5$ diversity branches are used, the probability that at least a single branch has instantaneous SNR greater than $\gamma$. [3 marks]

(iv) $M$, the number of diversity branches required, in order to sustain an outage probability of $10^{-5}$. [2 marks]

**Continued .....**

## Question 5

(a) With the aid of a diagram, briefly describe the *hidden terminal problem* in wireless networking and suggest a strategy to resolve this issue. [5 marks]

(b) (i) Between Frequency Division Multiplexing (FDM) and Orthogonal Frequency Division Multiplexing (OFDM), which multiplexing scheme is more spectral efficient? Justify your answer with the aid of diagrams. [4 marks]

(ii) With the aid of a frequency spectrum diagram, distinguish orthogonal frequency-division multiple access (OFDMA) from OFDM and state one application of OFDMA in modern wireless communication systems. [5 marks]

(c) Fig Q5 shows the logical separation of a Frequency Division Multiple Access/Frequency Division Duplex (FDMA/FDD) transmission system.

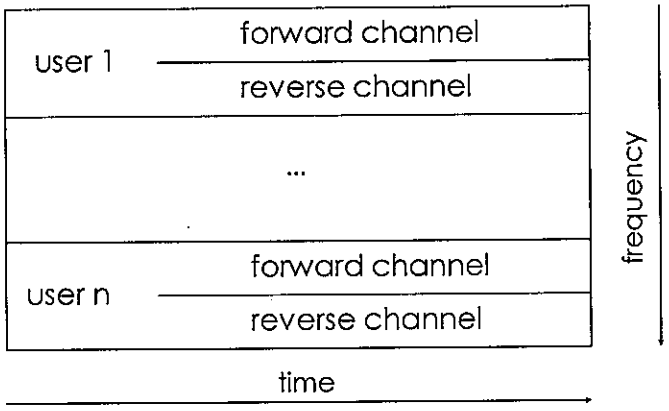


Fig. Q5

Using similar format, draw the logical separation diagram of the following transmission systems:

(i) Frequency Division Multiple Access/Time Division Duplex (FDMA/TDD). [2 marks]

(ii) Time Division Multiple Access/Time Division Duplex (TDMA/TDD). [2 marks]

(iii) Code Division Multiple Access/Time Division Duplex (CDMA/TDD). [2 marks]

**End of Questions**

## Appendix: Erlang B Table

### Erlang B Traffic Table

Maximum Offered Load Versus B and N
B is in %

| N/B | 0.01 | 0.05 | 0.1 | 0.5 | 1.0 | 2 | 5 | 10 | 15 | 20 | 30 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .0001 | .0005 | .0010 | .0050 | .0101 | .0204 | .0526 | .1111 | .1765 | .2500 | .4286 | .6667 |
| 2 | .0142 | .0321 | .0458 | .1054 | .1526 | .2235 | .3813 | .5954 | .7962 | 1.000 | 1.449 | 2.000 |
| 3 | .0868 | .1517 | .1938 | .3490 | .4555 | .6022 | .8994 | 1.271 | 1.603 | 1.930 | 2.633 | 3.480 |
| 4 | .2347 | .3624 | .4393 | .7012 | .8694 | 1.092 | 1.525 | 2.045 | 2.501 | 2.945 | 3.891 | 5.021 |
| 5 | .4520 | .6486 | .7621 | 1.132 | 1.361 | 1.657 | 2.219 | 2.881 | 3.454 | 4.010 | 5.189 | 6.596 |
| 6 | .7282 | .9957 | 1.146 | 1.622 | 1.909 | 2.276 | 2.960 | 3.758 | 4.445 | 5.109 | 6.514 | 8.191 |
| 7 | 1.054 | 1.392 | 1.579 | 2.158 | 2.501 | 2.935 | 3.738 | 4.666 | 5.461 | 6.230 | 7.856 | 9.800 |
| 8 | 1.422 | 1.830 | 2.051 | 2.730 | 3.128 | 3.627 | 4.543 | 5.597 | 6.498 | 7.369 | 9.213 | 11.42 |
| 9 | 1.826 | 2.302 | 2.558 | 3.333 | 3.783 | 4.345 | 5.370 | 6.546 | 7.551 | 8.522 | 10.58 | 13.05 |
| 10 | 2.260 | 2.803 | 3.092 | 3.961 | 4.461 | 5.084 | 6.216 | 7.511 | 8.616 | 9.685 | 11.95 | 14.68 |
| 11 | 2.722 | 3.329 | 3.651 | 4.610 | 5.160 | 5.842 | 7.076 | 8.487 | 9.691 | 10.86 | 13.33 | 16.31 |
| 12 | 3.207 | 3.878 | 4.231 | 5.279 | 5.876 | 6.615 | 7.950 | 9.474 | 10.78 | 12.04 | 14.72 | 17.95 |
| 13 | 3.713 | 4.447 | 4.831 | 5.964 | 6.607 | 7.402 | 8.835 | 10.47 | 11.87 | 13.22 | 16.11 | 19.60 |
| 14 | 4.239 | 5.032 | 5.446 | 6.663 | 7.352 | 8.200 | 9.730 | 11.47 | 12.97 | 14.41 | 17.50 | 21.24 |
| 15 | 4.781 | 5.634 | 6.077 | 7.376 | 8.108 | 9.010 | 10.63 | 12.48 | 14.07 | 15.61 | 18.90 | 22.89 |
| 16 | 5.339 | 6.250 | 6.722 | 8.100 | 8.875 | 9.828 | 11.54 | 13.50 | 15.18 | 16.81 | 20.30 | 24.54 |
| 17 | 5.911 | 6.878 | 7.378 | 8.834 | 9.652 | 10.66 | 12.46 | 14.52 | 16.29 | 18.01 | 21.70 | 26.19 |
| 18 | 6.496 | 7.519 | 8.046 | 9.578 | 10.44 | 11.49 | 13.39 | 15.55 | 17.41 | 19.22 | 23.10 | 27.84 |
| 19 | 7.093 | 8.170 | 8.724 | 10.33 | 11.23 | 12.33 | 14.32 | 16.58 | 18.53 | 20.42 | 24.51 | 29.50 |
| 20 | 7.701 | 8.831 | 9.412 | 11.09 | 12.03 | 13.18 | 15.25 | 17.61 | 19.65 | 21.64 | 25.92 | 31.15 |
| 21 | 8.319 | 9.501 | 10.11 | 11.86 | 12.84 | 14.04 | 16.19 | 18.65 | 20.77 | 22.85 | 27.33 | 32.81 |
| 22 | 8.946 | 10.18 | 10.81 | 12.64 | 13.65 | 14.90 | 17.13 | 19.69 | 21.90 | 24.06 | 28.74 | 34.46 |
| 23 | 9.583 | 10.87 | 11.52 | 13.42 | 14.47 | 15.76 | 18.08 | 20.74 | 23.03 | 25.28 | 30.15 | 36.12 |
| 24 | 10.23 | 11.56 | 12.24 | 14.20 | 15.30 | 16.63 | 19.03 | 21.78 | 24.16 | 26.50 | 31.56 | 37.78 |
| 25 | 10.88 | 12.26 | 12.97 | 15.00 | 16.13 | 17.51 | 19.99 | 22.83 | 25.30 | 27.72 | 32.97 | 39.44 |
| 26 | 11.54 | 12.97 | 13.70 | 15.80 | 16.96 | 18.38 | 20.94 | 23.89 | 26.43 | 28.94 | 34.39 | 41.10 |
| 27 | 12.21 | 13.69 | 14.44 | 16.60 | 17.80 | 19.27 | 21.90 | 24.94 | 27.57 | 30.16 | 35.80 | 42.76 |
| 28 | 12.88 | 14.41 | 15.18 | 17.41 | 18.64 | 20.15 | 22.87 | 26.00 | 28.71 | 31.39 | 37.21 | 44.41 |
| 29 | 13.56 | 15.13 | 15.93 | 18.22 | 19.49 | 21.04 | 23.83 | 27.05 | 29.85 | 32.61 | 38.63 | 46.07 |
| 30 | 14.25 | 15.86 | 16.68 | 19.03 | 20.34 | 21.93 | 24.80 | 28.11 | 31.00 | 33.84 | 40.05 | 47.74 |
| 31 | 14.94 | 16.60 | 17.44 | 19.85 | 21.19 | 22.83 | 25.77 | 29.17 | 32.14 | 35.07 | 41.46 | 49.40 |
| 32 | 15.63 | 17.34 | 18.21 | 20.68 | 22.05 | 23.73 | 26.75 | 30.24 | 33.28 | 36.30 | 42.88 | 51.06 |
| 33 | 16.34 | 18.09 | 18.97 | 21.51 | 22.91 | 24.63 | 27.72 | 31.30 | 34.43 | 37.52 | 44.30 | 52.72 |
| 34 | 17.04 | 18.84 | 19.74 | 22.34 | 23.77 | 25.53 | 28.70 | 32.37 | 35.58 | 38.75 | 45.72 | 54.38 |
| 35 | 17.75 | 19.59 | 20.52 | 23.17 | 24.64 | 26.44 | 29.68 | 33.43 | 36.72 | 39.99 | 47.14 | 56.04 |
| 36 | 18.47 | 20.35 | 21.30 | 24.01 | 25.51 | 27.34 | 30.66 | 34.50 | 37.87 | 41.22 | 48.56 | 57.70 |
| 37 | 19.19 | 21.11 | 22.08 | 24.85 | 26.38 | 28.25 | 31.64 | 35.57 | 39.02 | 42.45 | 49.98 | 59.37 |
| 38 | 19.91 | 21.87 | 22.86 | 25.69 | 27.25 | 29.17 | 32.62 | 36.64 | 40.17 | 43.68 | 51.40 | 61.03 |
| 39 | 20.64 | 22.64 | 23.65 | 26.53 | 28.13 | 30.08 | 33.61 | 37.72 | 41.32 | 44.91 | 52.82 | 62.69 |
| 40 | 21.37 | 23.41 | 24.44 | 27.38 | 29.01 | 31.00 | 34.60 | 38.79 | 42.48 | 46.15 | 54.24 | 64.35 |
| 41 | 22.11 | 24.19 | 25.24 | 28.23 | 29.89 | 31.92 | 35.58 | 39.86 | 43.63 | 47.38 | 55.66 | 66.02 |
| 42 | 22.85 | 24.97 | 26.04 | 29.09 | 30.77 | 32.84 | 36.57 | 40.94 | 44.78 | 48.62 | 57.08 | 67.68 |
| 43 | 23.59 | 25.75 | 26.84 | 29.94 | 31.66 | 33.76 | 37.57 | 42.01 | 45.94 | 49.85 | 58.50 | 69.34 |

(Continued on the next page)

## Appendix 1: Erlang B Table (continued)

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 44 | 24.33 | 26.53 | 27.64 | 30.80 | 32.54 | 34.68 | 38.56 | 43.09 | 47.09 | 51.09 | 59.92 | 71.01 |
| 45 | 25.08 | 27.32 | 28.45 | 31.66 | 33.43 | 35.61 | 39.55 | 44.17 | 48.25 | 52.32 | 61.35 | 72.67 |
| 46 | 25.83 | 28.11 | 29.26 | 32.52 | 34.32 | 36.53 | 40.55 | 45.24 | 49.40 | 53.56 | 62.77 | 74.33 |
| 47 | 26.59 | 28.90 | 30.07 | 33.38 | 35.22 | 37.46 | 41.54 | 46.32 | 50.56 | 54.80 | 64.19 | 76.00 |
| 48 | 27.34 | 29.70 | 30.88 | 34.25 | 36.11 | 38.39 | 42.54 | 47.40 | 51.71 | 56.03 | 65.61 | 77.66 |
| 49 | 28.10 | 30.49 | 31.69 | 35.11 | 37.00 | 39.32 | 43.53 | 48.48 | 52.87 | 57.27 | 67.04 | 79.32 |
| 50 | 28.87 | 31.29 | 32.51 | 35.98 | 37.90 | 40.26 | 44.53 | 49.56 | 54.03 | 58.51 | 68.46 | 80.99 |
| 51 | 29.63 | 32.09 | 33.33 | 36.85 | 38.80 | 41.19 | 45.53 | 50.64 | 55.19 | 59.75 | 69.88 | 82.65 |
| 52 | 30.40 | 32.90 | 34.15 | 37.72 | 39.70 | 42.12 | 46.53 | 51.73 | 56.35 | 60.99 | 71.31 | 84.32 |
| 53 | 31.17 | 33.70 | 34.98 | 38.60 | 40.60 | 43.06 | 47.53 | 52.81 | 57.50 | 62.22 | 72.73 | 85.98 |
| 54 | 31.94 | 34.51 | 35.80 | 39.47 | 41.51 | 44.00 | 48.54 | 53.89 | 58.66 | 63.46 | 74.15 | 87.65 |
| 55 | 32.72 | 35.32 | 36.63 | 40.35 | 42.41 | 44.94 | 49.54 | 54.98 | 59.82 | 64.70 | 75.58 | 89.31 |
| 56 | 33.49 | 36.13 | 37.46 | 41.23 | 43.32 | 45.88 | 50.54 | 56.06 | 60.98 | 65.94 | 77.00 | 90.97 |
| 57 | 34.27 | 36.95 | 38.29 | 42.11 | 44.22 | 46.82 | 51.55 | 57.14 | 62.14 | 67.18 | 78.43 | 92.64 |
| 58 | 35.05 | 37.76 | 39.12 | 42.99 | 45.13 | 47.76 | 52.55 | 58.23 | 63.31 | 68.42 | 79.85 | 94.30 |
| 59 | 35.84 | 38.58 | 39.96 | 43.87 | 46.04 | 48.70 | 53.56 | 59.32 | 64.47 | 69.66 | 81.27 | 95.97 |
| 60 | 36.62 | 39.40 | 40.80 | 44.76 | 46.95 | 49.64 | 54.57 | 60.40 | 65.63 | 70.90 | 82.70 | 97.63 |
| 61 | 37.41 | 40.22 | 41.63 | 45.64 | 47.86 | 50.59 | 55.57 | 61.49 | 66.79 | 72.14 | 84.12 | 99.30 |
| 62 | 38.20 | 41.05 | 42.47 | 46.53 | 48.77 | 51.53 | 56.58 | 62.58 | 67.95 | 73.38 | 85.55 | 101.0 |
| 63 | 38.99 | 41.87 | 43.31 | 47.42 | 49.69 | 52.48 | 57.59 | 63.66 | 69.11 | 74.63 | 86.97 | 102.6 |
| 64 | 39.78 | 42.70 | 44.16 | 48.31 | 50.60 | 53.43 | 58.60 | 64.75 | 70.28 | 75.87 | 88.40 | 104.3 |
| 65 | 40.58 | 43.52 | 45.00 | 49.20 | 51.52 | 54.38 | 59.61 | 65.84 | 71.44 | 77.11 | 89.82 | 106.0 |
| 66 | 41.38 | 44.35 | 45.85 | 50.09 | 52.44 | 55.33 | 60.62 | 66.93 | 72.60 | 78.35 | 91.25 | 107.6 |
| 67 | 42.17 | 45.18 | 46.69 | 50.98 | 53.35 | 56.28 | 61.63 | 68.02 | 73.77 | 79.59 | 92.67 | 109.3 |
| 68 | 42.97 | 46.02 | 47.54 | 51.87 | 54.27 | 57.23 | 62.64 | 69.11 | 74.93 | 80.83 | 94.10 | 111.0 |
| 69 | 43.77 | 46.85 | 48.39 | 52.77 | 55.19 | 58.18 | 63.65 | 70.20 | 76.09 | 82.08 | 95.52 | 112.6 |
| 70 | 44.58 | 47.68 | 49.24 | 53.66 | 56.11 | 59.13 | 64.67 | 71.29 | 77.26 | 83.32 | 96.95 | 114.3 |
| 71 | 45.38 | 48.52 | 50.09 | 54.56 | 57.03 | 60.08 | 65.68 | 72.38 | 78.42 | 84.56 | 98.37 | 116.0 |
| 72 | 46.19 | 49.36 | 50.94 | 55.46 | 57.96 | 61.04 | 66.69 | 73.47 | 79.59 | 85.80 | 99.80 | 117.6 |
| 73 | 47.00 | 50.20 | 51.80 | 56.35 | 58.88 | 61.99 | 67.71 | 74.56 | 80.75 | 87.05 | 101.2 | 119.3 |
| 74 | 47.81 | 51.04 | 52.65 | 57.25 | 59.80 | 62.95 | 68.72 | 75.65 | 81.92 | 88.29 | 102.7 | 120.9 |
| 75 | 48.62 | 51.88 | 53.51 | 58.15 | 60.73 | 63.90 | 69.74 | 76.74 | 83.08 | 89.53 | 104.1 | 122.6 |
| 76 | 49.43 | 52.72 | 54.37 | 59.05 | 61.65 | 64.86 | 70.75 | 77.83 | 84.25 | 90.78 | 105.5 | 124.3 |
| 77 | 50.24 | 53.56 | 55.23 | 59.96 | 62.58 | 65.81 | 71.77 | 78.93 | 85.41 | 92.02 | 106.9 | 125.9 |
| 78 | 51.05 | 54.41 | 56.09 | 60.86 | 63.51 | 66.77 | 72.79 | 80.02 | 86.58 | 93.26 | 108.4 | 127.6 |
| 79 | 51.87 | 55.25 | 56.95 | 61.76 | 64.43 | 67.73 | 73.80 | 81.11 | 87.74 | 94.51 | 109.8 | 129.3 |
| 80 | 52.69 | 56.10 | 57.81 | 62.67 | 65.36 | 68.69 | 74.82 | 82.20 | 88.91 | 95.75 | 111.2 | 130.9 |
| 81 | 53.51 | 56.95 | 58.67 | 63.57 | 66.29 | 69.65 | 75.84 | 83.30 | 90.08 | 96.99 | 112.6 | 132.6 |
| 82 | 54.33 | 57.80 | 59.54 | 64.48 | 67.22 | 70.61 | 76.86 | 84.39 | 91.24 | 98.24 | 114.1 | 134.3 |
| 83 | 55.15 | 58.65 | 60.40 | 65.39 | 68.15 | 71.57 | 77.87 | 85.48 | 92.41 | 99.48 | 115.5 | 135.9 |
| 84 | 55.97 | 59.50 | 61.27 | 66.29 | 69.08 | 72.53 | 78.89 | 86.58 | 93.58 | 100.7 | 116.9 | 137.6 |
| 85 | 56.79 | 60.35 | 62.14 | 67.20 | 70.02 | 73.49 | 79.91 | 87.67 | 94.74 | 102.0 | 118.3 | 139.3 |
| 86 | 57.62 | 61.21 | 63.00 | 68.11 | 70.95 | 74.45 | 80.93 | 88.77 | 95.91 | 103.2 | 119.8 | 140.9 |
| 87 | 58.44 | 62.06 | 63.87 | 69.02 | 71.88 | 75.42 | 81.95 | 89.86 | 97.08 | 104.5 | 121.2 | 142.6 |
| 88 | 59.27 | 62.92 | 64.74 | 69.93 | 72.82 | 76.38 | 82.97 | 90.96 | 98.25 | 105.7 | 122.6 | 144.3 |
| 89 | 60.10 | 63.77 | 65.61 | 70.84 | 73.75 | 77.34 | 83.99 | 92.05 | 99.41 | 107.0 | 124.0 | 145.9 |
| 90 | 60.92 | 64.63 | 66.48 | 71.76 | 74.68 | 78.31 | 85.01 | 93.15 | 100.6 | 108.2 | 125.5 | 147.6 |

## Appendix 2: Tabulated values of Q-function

| Q(x) x | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 5.000E-01 | 4.960E-01 | 4.920E-01 | 4.880E-01 | 4.840E-01 | 4.801E-01 | 4.761E-01 | 4.721E-01 | 4.681E-01 | 4.641E-01 |
| 0.10 | 4.602E-01 | 4.562E-01 | 4.522E-01 | 4.483E-01 | 4.443E-01 | 4.404E-01 | 4.364E-01 | 4.325E-01 | 4.286E-01 | 4.247E-01 |
| 0.20 | 4.207E-01 | 4.168E-01 | 4.129E-01 | 4.090E-01 | 4.052E-01 | 4.013E-01 | 3.974E-01 | 3.936E-01 | 3.897E-01 | 3.859E-01 |
| 0.30 | 3.821E-01 | 3.783E-01 | 3.745E-01 | 3.707E-01 | 3.669E-01 | 3.632E-01 | 3.594E-01 | 3.557E-01 | 3.520E-01 | 3.483E-01 |
| 0.40 | 3.446E-01 | 3.409E-01 | 3.372E-01 | 3.336E-01 | 3.300E-01 | 3.264E-01 | 3.228E-01 | 3.192E-01 | 3.156E-01 | 3.121E-01 |
| 0.50 | 3.085E-01 | 3.050E-01 | 3.015E-01 | 2.981E-01 | 2.946E-01 | 2.912E-01 | 2.877E-01 | 2.843E-01 | 2.810E-01 | 2.776E-01 |
| 0.60 | 2.743E-01 | 2.709E-01 | 2.676E-01 | 2.643E-01 | 2.611E-01 | 2.578E-01 | 2.546E-01 | 2.514E-01 | 2.483E-01 | 2.451E-01 |
| 0.70 | 2.420E-01 | 2.389E-01 | 2.358E-01 | 2.327E-01 | 2.296E-01 | 2.266E-01 | 2.236E-01 | 2.206E-01 | 2.177E-01 | 2.148E-01 |
| 0.80 | 2.119E-01 | 2.090E-01 | 2.061E-01 | 2.033E-01 | 2.005E-01 | 1.977E-01 | 1.949E-01 | 1.922E-01 | 1.894E-01 | 1.867E-01 |
| 0.90 | 1.841E-01 | 1.814E-01 | 1.788E-01 | 1.762E-01 | 1.736E-01 | 1.711E-01 | 1.685E-01 | 1.660E-01 | 1.635E-01 | 1.611E-01 |
| 1.00 | 1.587E-01 | 1.562E-01 | 1.539E-01 | 1.515E-01 | 1.492E-01 | 1.469E-01 | 1.446E-01 | 1.423E-01 | 1.401E-01 | 1.379E-01 |
| 1.10 | 1.357E-01 | 1.335E-01 | 1.314E-01 | 1.292E-01 | 1.271E-01 | 1.251E-01 | 1.230E-01 | 1.210E-01 | 1.190E-01 | 1.170E-01 |
| 1.20 | 1.151E-01 | 1.131E-01 | 1.112E-01 | 1.093E-01 | 1.075E-01 | 1.056E-01 | 1.038E-01 | 1.020E-01 | 1.003E-01 | 9.853E-02 |
| 1.30 | 9.680E-02 | 9.510E-02 | 9.342E-02 | 9.176E-02 | 9.012E-02 | 8.851E-02 | 8.691E-02 | 8.534E-02 | 8.379E-02 | 8.226E-02 |
| 1.40 | 8.076E-02 | 7.927E-02 | 7.780E-02 | 7.636E-02 | 7.493E-02 | 7.353E-02 | 7.215E-02 | 7.078E-02 | 6.944E-02 | 6.811E-02 |
| 1.50 | 6.681E-02 | 6.552E-02 | 6.426E-02 | 6.301E-02 | 6.178E-02 | 6.057E-02 | 5.938E-02 | 5.821E-02 | 5.705E-02 | 5.592E-02 |
| 1.60 | 5.480E-02 | 5.370E-02 | 5.262E-02 | 5.155E-02 | 5.050E-02 | 4.947E-02 | 4.846E-02 | 4.746E-02 | 4.648E-02 | 4.551E-02 |
| 1.70 | 4.457E-02 | 4.363E-02 | 4.272E-02 | 4.182E-02 | 4.093E-02 | 4.006E-02 | 3.920E-02 | 3.836E-02 | 3.754E-02 | 3.673E-02 |
| 1.80 | 3.593E-02 | 3.515E-02 | 3.438E-02 | 3.362E-02 | 3.288E-02 | 3.216E-02 | 3.144E-02 | 3.074E-02 | 3.005E-02 | 2.938E-02 |
| 1.90 | 2.872E-02 | 2.807E-02 | 2.743E-02 | 2.680E-02 | 2.619E-02 | 2.559E-02 | 2.500E-02 | 2.442E-02 | 2.385E-02 | 2.330E-02 |
| 2.00 | 2.275E-02 | 2.222E-02 | 2.169E-02 | 2.118E-02 | 2.068E-02 | 2.018E-02 | 1.970E-02 | 1.923E-02 | 1.876E-02 | 1.831E-02 |
| 2.10 | 1.786E-02 | 1.743E-02 | 1.700E-02 | 1.659E-02 | 1.618E-02 | 1.578E-02 | 1.539E-02 | 1.500E-02 | 1.463E-02 | 1.426E-02 |
| 2.20 | 1.390E-02 | 1.355E-02 | 1.321E-02 | 1.287E-02 | 1.255E-02 | 1.222E-02 | 1.191E-02 | 1.160E-02 | 1.130E-02 | 1.101E-02 |
| 2.30 | 1.072E-02 | 1.044E-02 | 1.017E-02 | 9.903E-03 | 9.642E-03 | 9.387E-03 | 9.137E-03 | 8.894E-03 | 8.656E-03 | 8.424E-03 |
| 2.40 | 8.198E-03 | 7.976E-03 | 7.760E-03 | 7.549E-03 | 7.344E-03 | 7.143E-03 | 6.947E-03 | 6.756E-03 | 6.569E-03 | 6.387E-03 |
| 2.50 | 6.210E-03 | 6.037E-03 | 5.868E-03 | 5.703E-03 | 5.543E-03 | 5.386E-03 | 5.234E-03 | 5.085E-03 | 4.940E-03 | 4.799E-03 |
| 2.60 | 4.661E-03 | 4.527E-03 | 4.396E-03 | 4.269E-03 | 4.145E-03 | 4.025E-03 | 3.907E-03 | 3.793E-03 | 3.681E-03 | 3.573E-03 |
| 2.70 | 3.467E-03 | 3.364E-03 | 3.264E-03 | 3.167E-03 | 3.072E-03 | 2.980E-03 | 2.890E-03 | 2.803E-03 | 2.718E-03 | 2.635E-03 |
| 2.80 | 2.555E-03 | 2.477E-03 | 2.401E-03 | 2.327E-03 | 2.256E-03 | 2.186E-03 | 2.118E-03 | 2.052E-03 | 1.988E-03 | 1.926E-03 |
| 2.90 | 1.866E-03 | 1.807E-03 | 1.750E-03 | 1.695E-03 | 1.641E-03 | 1.589E-03 | 1.538E-03 | 1.489E-03 | 1.441E-03 | 1.395E-03 |
| 3.00 | 1.350E-03 | 1.306E-03 | 1.264E-03 | 1.223E-03 | 1.183E-03 | 1.144E-03 | 1.107E-03 | 1.070E-03 | 1.035E-03 | 1.001E-03 |
| 3.10 | 9.676E-04 | 9.354E-04 | 9.043E-04 | 8.740E-04 | 8.447E-04 | 8.164E-04 | 7.888E-04 | 7.622E-04 | 7.364E-04 | 7.114E-04 |
| 3.20 | 6.871E-04 | 6.637E-04 | 6.410E-04 | 6.190E-04 | 5.976E-04 | 5.770E-04 | 5.571E-04 | 5.377E-04 | 5.190E-04 | 5.009E-04 |
| 3.30 | 4.834E-04 | 4.665E-04 | 4.501E-04 | 4.342E-04 | 4.189E-04 | 4.041E-04 | 3.897E-04 | 3.758E-04 | 3.624E-04 | 3.495E-04 |
| 3.40 | 3.369E-04 | 3.248E-04 | 3.131E-04 | 3.018E-04 | 2.909E-04 | 2.803E-04 | 2.701E-04 | 2.602E-04 | 2.507E-04 | 2.415E-04 |
| 3.50 | 2.326E-04 | 2.241E-04 | 2.158E-04 | 2.078E-04 | 2.001E-04 | 1.926E-04 | 1.854E-04 | 1.785E-04 | 1.718E-04 | 1.653E-04 |
| 3.60 | 1.591E-04 | 1.531E-04 | 1.473E-04 | 1.417E-04 | 1.363E-04 | 1.311E-04 | 1.261E-04 | 1.213E-04 | 1.166E-04 | 1.121E-04 |
| 3.70 | 1.078E-04 | 1.036E-04 | 9.961E-05 | 9.574E-05 | 9.201E-05 | 8.842E-05 | 8.496E-05 | 8.162E-05 | 7.841E-05 | 7.532E-05 |
| 3.80 | 7.235E-05 | 6.948E-05 | 6.673E-05 | 6.407E-05 | 6.152E-05 | 5.906E-05 | 5.669E-05 | 5.442E-05 | 5.223E-05 | 5.012E-05 |
| 3.90 | 4.810E-05 | 4.615E-05 | 4.427E-05 | 4.247E-05 | 4.074E-05 | 3.908E-05 | 3.747E-05 | 3.594E-05 | 3.446E-05 | 3.304E-05 |
| 4.00 | 3.167E-05 | 3.036E-05 | 2.910E-05 | 2.789E-05 | 2.673E-05 | 2.561E-05 | 2.454E-05 | 2.351E-05 | 2.252E-05 | 2.157E-05 |
| 4.10 | 2.066E-05 | 1.978E-05 | 1.894E-05 | 1.814E-05 | 1.737E-05 | 1.662E-05 | 1.591E-05 | 1.523E-05 | 1.458E-05 | 1.395E-05 |
| 4.20 | 1.335E-05 | 1.277E-05 | 1.222E-05 | 1.168E-05 | 1.118E-05 | 1.069E-05 | 1.022E-05 | 9.774E-06 | 9.345E-06 | 8.934E-06 |
| 4.30 | 8.540E-06 | 8.163E-06 | 7.801E-06 | 7.455E-06 | 7.124E-06 | 6.807E-06 | 6.503E-06 | 6.212E-06 | 5.934E-06 | 5.668E-06 |
| 4.40 | 5.413E-06 | 5.169E-06 | 4.935E-06 | 4.712E-06 | 4.498E-06 | 4.294E-06 | 4.098E-06 | 3.911E-06 | 3.732E-06 | 3.561E-06 |
| 4.50 | 3.398E-06 | 3.241E-06 | 3.092E-06 | 2.949E-06 | 2.813E-06 | 2.682E-06 | 2.558E-06 | 2.439E-06 | 2.325E-06 | 2.216E-06 |
| 4.60 | 2.112E-06 | 2.013E-06 | 1.919E-06 | 1.828E-06 | 1.742E-06 | 1.660E-06 | 1.581E-06 | 1.506E-06 | 1.434E-06 | 1.366E-06 |
| 4.70 | 1.301E-06 | 1.239E-06 | 1.179E-06 | 1.123E-06 | 1.069E-06 | 1.017E-06 | 9.680E-07 | 9.211E-07 | 8.765E-07 | 8.339E-07 |
| 4.80 | 7.933E-07 | 7.547E-07 | 7.178E-07 | 6.827E-07 | 6.492E-07 | 6.173E-07 | 5.869E-07 | 5.580E-07 | 5.304E-07 | 5.042E-07 |
| 4.90 | 4.792E-07 | 4.554E-07 | 4.327E-07 | 4.111E-07 | 3.906E-07 | 3.711E-07 | 3.525E-07 | 3.348E-07 | 3.179E-07 | 3.019E-07 |

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